Name:	Target Grade:	Actual Grade:

BRIGHT CULTURE

INDICES

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Find a quiet, comfortable spot free place from distractions.
2. Spend one minute on each mark.
3. Time yourself for every single question.
4. Every chapter has their own question types. Ensure that you know the different question type for each chapter.
5. Make a conscientious effort to remember your mistakes, especially in terms of answering techniques. E.g Take a picture for the mistakes that you made, keep it in a photo album, and revise it over and over again.
6. Highlight question types that you tend to keep making mistakes and review them nearing exams.
7. Always review the common questions and question type that you tend to make mistakes nearing exams.
8. During exams, classify the question type and recall what you have learnt, how you need to analvse the questions for the different question type, what you need to take note of and answer with the correct answering techniques!

✨ Wishing you all the best for this test!

You've got this!

💡 With lots of love,
Bright Culture 💛

MARKS

If you are struggling in this paper, means you need to work harder!

If you need any professional guidance and further advice on how to advance, feel free to WhatsApp us at 91870820 or find us at www.bright-culture.com/. We are committed to connect you to your future to reach your goals.

CHAPTER 3: INDICES

1 (a) Simplify $\left(\dfrac{x^6}{25y^4}\right)^{-\frac{1}{2}}$, giving your answer in positive indices.

Answer ……………………….. [2]

(b) Solve the equation $9\sqrt[3]{3^{3x}} = \dfrac{1}{3^{3(2-x)}}$.

Answer $x =$ ……………………….. [3]

(c) Given that $a > 0$ and n is an even number, deduce the number of solutions for the equation $ax^n - x = 0$. Explain your answer clearly.

Answer ……………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

…………………………………………………………………………………… [3]

2 (a) It is given that $\frac{2px+9qy}{2py+qx}=3$, p and q are constants and $2p \neq 3q$.

(i) Show that $x = 3y$.

[2]

(ii) Evaluate $\frac{x+y}{y}$.

Answer ______________________________[2]

(b) Simplify $\frac{3a+7b}{16a^2-49(a+b)^2}$.

Answer ______________________________[3]

(c) Solve the equation $2^{x+3} = 320 - 2^{x+1}$.

Answer x = ______________________________ [3]

3 Show that $3^{3x+2} - 9^{\frac{3}{2}x} + (27)^{x+1}$ is divisible by 5 for all positive integer values.

Answer

[2]

4 (a) (i) Factorise $2px - 2p + 3qx - 3q$ completely.

Answer …………………………….. [2]

(ii) Given that p and q are positive constants, find the value of x for which $2px - 2p + 3qx - 3q = 0$.

Answer $x =$ …………………………….. [1]

(b) Simplify $\left(-3p^2q^{-1}\right)^2\left(p^{-2}q^2\right)^3$, expressing your final answer in positive indices.

Answer …………………………….. [2]

5 (a) Simplify $\frac{2x}{x^0} \div \left(\frac{2y}{x}\right)^{-2}$, leaving your answer in positive indices.

Answer ____________________________________ [3]

(b) Express y in terms of p and q, given that $\frac{1}{q} = \frac{2}{y} + p$.

Answer ____________________________________ [3]

6 Simplify $\left(\dfrac{p^6}{9q^4}\right)^{-\frac{1}{2}}$.

Answer .... [2]

7 (a) Simplify

(i) $\dfrac{3a}{b^3} \div \dfrac{9a^2}{(2b)^3}$,

Answer ______________________________ [1]

(ii) $\dfrac{50-8y^2}{4y^2-6y-10}$.

Answer ______________________________ [3]

(b) Write as a single fraction in its simplest form $\dfrac{3}{6x-2}+\dfrac{2}{(3x-1)^2}$.

Answer ______________________________ [2]

(c) Solve $\dfrac{4}{x+1}-1=\dfrac{9}{2x+7}$.

Answer $x =$ ……………………….. or ……………………. [3]

8 (a) Expressing your answer as a power of 6, find $6^5 \div 6^{-3} \times 6^3$.

Answer ……………………………. [1]

(b) Simplify $\dfrac{(3x^2)^3}{21x^4} \times 5x^{-2} + 7x^0$.

Answer ……………………………. [2]

9 (a) Simplify $\left(\frac{p^6}{q^3}\right)^{-\frac{2}{3}}$, leaving your answer in positive indices.

Answer …………………………. [2]

(b) Given that $8^{1+b} = 16^{b-2}$, find the value of b.

Answer b …………………………. [2]

10 Without using a calculator, show that $5^{2018} - 5^{2017}$ is an even number.

Answer

[2]

11 Given that $\frac{8^{-1}}{8^{k}} \times 64 = 1$, find k.

Answer k = ____________________ [2]

12 (a) Solve the inequality $\frac{3-x}{2} \le 2 - \frac{x}{3}$.

Answer …………………………… [2]

(b) Express as a single fraction in its simplest form $\frac{3y}{(3-2y)^2} + \frac{y}{(2y-3)}$.

Answer …………………………… [2]

(c) Simplify $\frac{6p^2q}{15r^3} \div \frac{3p^3q^2}{5r} \times \frac{p^4}{q^2}$, leave your answer in positive indices.

Answer …………………………… [2]

(d) Simplify $\left(\frac{v^{12}}{27t^9}\right)^{-\frac{2}{3}}$, **leave your answer in positive indices.**

Answer …………………………… [2]

(e) Solve the equation $\frac{8}{x} - \frac{3}{x+1} = 3$.

Answer $x =$ …………… *or* ……………… [3]

13 The following table shows the world population by region in 2020.

Region	**Population**
Asia	4.64×10^9
Africa	1.34×10^9
Europe	7.48×10^8
Southern America	6.54×10^8
Northern America	3.69×10^8
Oceania	42,677,813

(a) Express the population of Oceania in standard form, correct to 3 significant figures.

Answer ……………………………… [1]

(b) Calculate the percentage of population in Asia compared to the world population.

Answer ………………………………% [2]

(c) From 2019 to 2020, the total population of Japan decreased by 1.3%. The population of Japan in 2020 is 1.261×10^8 . Find the population of Japan in 2019. Give your answer in standard form.

Answer ……………………………… [2]

(d) The population of Africa grew by 2.5% every year for two consecutive years from 2018 to 2020. Calculate the population of Africa in 2018. Give your answer in standard form.

Answer ……………………………… [2]

14 Simplify $\left(\dfrac{81}{y^8}\right)^{-\frac{1}{4}} \times \left(3y^2\right)^3$.

Answer …………………………… [2]

15 (a) Given that $2\times 5^x = \dfrac{10}{\sqrt[3]{25}}$, find the value of *x*.

Answer x = …………………………… [2]

(b) The radius of the base of a cylinder was increased by 30% and its height was decreased by 30%.

Find the percentage change, if any, in its volume.

Answer …………………………… % [3]

16 (a) Express $\dfrac{4}{2-3x}+\dfrac{4(x+7)}{21x^2-5x-6}$ as a single fraction in its simplest form.

Answer ……………………………… [3]

(b) Simplify $\left(\dfrac{81p^4}{q^8}\right)^{\frac{1}{4}} \div \left(\dfrac{r^3}{3q^2}\right)^{-5}$, leaving your answer in positive index.

Answer ……………………………… [3]

(c) Given that $m = k\sqrt{\dfrac{3n-7}{4}}$, express n in terms of k and m.

Answer ……………………………… [2]

17 Simplify

(a) $a^2b \times a^{-2}b^5$,

Answer ……………………………….. [1]

(b) $\left(\dfrac{27}{x^{18}}\right)^{\frac{-1}{3}}$.

Answer ……………………………….. [2]

18 (a) Simplify $\left(\dfrac{y^3}{6x}\right)^{-1} \times (xy^2)^2$

Answer (a) ……………………………….. [2]

(b) Given that $2^x = 5$, find the value of

(i) 8^x,

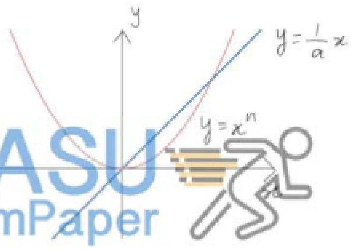
Answer (b)(i) ……………………………….. [1]

(ii) 2^{1-x}.

Answer (b)(ii) ……………………………….. [1]

ANSWERS

Qn	Solution	Marks	Total Marks
1a	$\left(\frac{x^6}{25y^4}\right)^{-\frac{1}{2}} = \left(\frac{25y^4}{x^6}\right)^{\frac{1}{2}}$ $= \frac{5y^2}{x^3}$ **Alternative Method:** $\left(\frac{x^6}{25y^4}\right)^{-\frac{1}{2}} = \frac{x^{-3}}{5^{-1}y^{-2}}$ $= \frac{5y^2}{x^3}$	M1 A1 M1 A1	2
1b	$9\sqrt[3]{3^{3x}} = \frac{1}{3^{3(2-x)}}$ $3^2 \times 3^x = \frac{1}{3^{6-3x}}$ $3^{2+x} = 3^{-6+3x}$ $2+x=-6+3x$ $-2x=-8$ $x=4$	M1 M1 A1	3

1c	$x(ax^{n-1}-1)=0$ $x=0$ or $ax^{n-1}-1=0$ When $ax^{n-1}-1=0$, $ax^{n-1}=1$ If n is even, $n-1$ is odd. Hence, $ax^{n-1}=1$ will have 1 solution. There will be a total of 2 solutions for the given equation. **Alternative Method:** $ax^n=x$ $x^n=\frac{1}{a}x$ When n is even, we will have a curve and a straight line as seen below.  Hence, there will be 2 solutions for the given equation.	M1 M1 A1 M1, M1 (1 mark for each sketch) A1	3

2	(a)(i)	$\frac{2px+9qy}{2py+qx}=3$ $2px+9qy=6py+3qx$ $2px-3qx=6py-9qy$ $x(2p-3q)=3y(2p-3q)$ Since $2p\neq 3q$, $2p-3q\neq 0$ $x=3y$ (shown)	[M1] [A1]	
	(a)(ii)	$\frac{x}{y}=3$ $\frac{x+y}{y}=\frac{x}{y}+1$ $=3+1$ $=4$	[M1] [A1]	
	(b)	$\frac{3a+7b}{16a^2-49(a+b)^2}=\frac{3a+7b}{[4a+7(a+b)][4a-7(a+b)]}$ $=\frac{3a+7b}{(11a+7b)(-3a-7b)}$ $=-\frac{1}{11a+7b}$	[M1] [M1] [A1]	

	(c)	$2^{x+3}=320-2^{x+1}$ $2^{x+1}+2^{x+3}=320$ $2^x\times 2+2^x\times 2^3=320$ $2^x(2+8)=320$ $2^x=32$ $=2^5$ $x=\underline{\underline{5}}$	[M1] [M1] [A1]	
3	$3^{3x+2}-9^{\frac{3}{2}x}+(27)^{x+1}=3^{3x+2}-3^{3x}+3^{3x+3}$ $=3^{3x}(3^2-1+3^3)$ $=3^{3x}(35)$ Since 35 is a multiple of 5, hence $3^{3x+2}-9^{\frac{3}{2}x}+\left(\frac{1}{27}\right)^{-x-1}$ is divisible by 5 for all positive integer values.		M1 A1	M – express all in index notation with base 3 Any multiple of 5
4	(a)(i)	$2px-2p+3qx-3q$ $=2p(x-1)+3q(x-1)$ $=(2p+3q)(x-1)$	M1 A1	Grouping ($2p+3q$)
	(ii)	$(2p+3q)(x-1)=0$ $(x-1)=0$ or (2p+3q)=0 [optional in this case.] $x=1$	B1	
	(b)	$(-3p^2q^{-1})^2(p^{-2}q^2)^3$ $=9p^4q^{-2}p^{-6}q^6$ $=9p^{-2}q^4$ $=\frac{9q^4}{p^2}$	M[1] (correct use of $(ab)^n=a^nb^n$) A[1]	
5	(a)	$\frac{2x}{x^0}\div\left(\frac{2y}{x}\right)^{-2}=\frac{2x}{1}\div\left(\frac{x}{2y}\right)^2$ $=\frac{2x}{1}\div\frac{x^2}{4y^2}$ $=\frac{2x}{1}\times\frac{4y^2}{x^2}$ $=\frac{8y^2}{x}$	[M1] [M1] [A1]	

	(b)	$\frac{1}{q}=\frac{2}{y}+p$ $\frac{2}{y}=\frac{1}{q}-p$ $\frac{2}{y}=\frac{1}{q}-\frac{pq}{q}$ $\frac{2}{y}=\frac{1-pq}{q}$ $\frac{y}{2}=\frac{q}{1-pq}$ $y=\frac{2q}{1-pq}$	[M1] [M1] [M1]
6		$(3q^2)/p^3$	
7		(a) $\frac{8}{3a}$ (b) $\frac{-(5+2y)}{y+1}$ (c) $\frac{9x+1}{2(3x-1)^2}$ (d) $x=1$ or -6	
8	(a)	$6^5 \div 6^{-3} \times 6^3 = 6^{5-(-3)+3}$ $= 6^{11}$	
	(b)	$\frac{(3x^2)^3}{21x^4} \times 5x^{-2} + 7x^0$ $= \frac{27x^6}{21x^4} \times \frac{5}{x^2} + 7$ $= 13\frac{3}{7}$	
9		$\left(\frac{p^6}{q^3}\right)^{-\frac{2}{3}} = \frac{p^{-4}}{q^{-2}}$ $= \frac{q^2}{p^4}$	
10	$5^{2018}-5^{2017}=5^{2017}(5-1)$ $=5^{2017}\times 4$ **Option 1:** Since 4 is an **<u>even factor</u>** of $5^{2018}-5^{2017}$, $5^{2018}-5^{2017}$ is an even number. **Option 2:** Since $5^{2018}-5^{2017}$ is a **<u>multiple of 4, which is an even number</u>**, $5^{2018}-5^{2017}$ is an even number. **Option 3:** Since **<u>4 is an even number</u>**, and an even number **<u>multiplied by any number is even</u>**, $5^{2018}-5^{2017}$ is an even number.		

11

$$\frac{8^{-1}}{8^k} \times 64 = 1$$

$$8^{-1-k} \times 8^2 = 8^0 \quad \text{M1}$$

$$8^{1-k} = 8^0$$

$$1 - k = 0$$

$$k = 1 \quad \text{A1}$$

12 (a)

$$\frac{3-x}{2} \le 2 - \frac{x}{3}$$

$$\frac{3-x}{2} \le \frac{6-x}{3} \quad \text{(M1)}$$

$$9 - 3x \le 12 - 2x$$

$$x \ge -3 \quad \text{(A1)}$$

(b)

$$\frac{3y}{(3-2y)^2} + \frac{y}{(2y-3)} = \frac{3y}{(3-2y)^2} - \frac{y(3-2y)}{(3-2y)^2} \quad \text{(M1)}$$

$$= \frac{3y - 3y + 2y^2}{(3-2y)^2}$$

$$= \frac{2y^2}{(3-2y)^2} \quad \text{(A1)}$$

(c)

$$\frac{6p^2q}{15r^3} \div \frac{3p^3q^2}{5r} \times \frac{p^4}{q^2} = \frac{6p^2q}{15r^3} \times \frac{5r}{3p^3q^2} \times \frac{p^4}{q^2} \quad \text{(M1)}$$

$$= \frac{2}{3} p^{2+4-3} q^{1-2-2} r^{1-3}$$

$$= \frac{2}{3} p^3 q^{-3} r^{-2}$$

$$= \frac{2p^3}{3q^3r^2} \quad \text{(A1)}$$

(d)

$$\left(\frac{v^{12}}{27t^9}\right)^{-\frac{2}{3}} = \left(\frac{27t^9}{v^{12}}\right)^{\frac{2}{3}} \quad \text{(M1)}$$

$$= \left(\frac{3^3t^9}{v^{12}}\right)^{\frac{2}{3}}$$

$$= \frac{9t^6}{v^8} \quad \text{(A1)}$$

(e)

$$\frac{8}{x}-\frac{3}{x+1}=3$$

$$\frac{8(x+1)-3x}{x(x+1)}=3 \qquad \text{(M1)}$$

$$\frac{5x+8}{x^2+x}=3$$

$$5x+8=3x^2+3x$$

$$3x^2-2x-8=0$$

$$(3x+4)(x-2)=0 \qquad \text{(M1)}$$

$$x=2 \text{ or } -1\frac{1}{3} \qquad \text{(A1)}$$

13 (a) 4.27×10^7 (A1)

(b) Percentage of population in Asia $=\dfrac{464\times10^7}{(464+134+74.8+65.4+36.9+4.27)\times10^7}\times100\%$ (M1)

$$=\frac{464\times10^7}{779.37\times10^7}\times100\%$$

$$=59.5\% \qquad \text{(A1)}$$

(c)

Population of Japan $=\dfrac{1.261\times10^8}{0.987}$ (M1)

$$=1.2776\times10^8$$

$$=1.28\times10^8 \qquad \text{(A1)}$$

(d) Population of Africa $=\dfrac{1.34\times10^9}{(1.025)^2}$ (M1)

$$=1.28\times10^9 \qquad \text{(A1)}$$

14 $9y^8$

15 (a)

$$2\times5^x=\frac{10}{\sqrt[3]{25}}$$

$$5^x=\frac{5}{\sqrt[3]{25}}$$

$$=5\times5^{-\frac{2}{3}}$$

$$5^x=5^{\frac{1}{3}}$$

By comparison,

$$x=\frac{1}{3}$$

(b) Let the radius of the cylinder be r cm and the height be h cm.

Original Volume $=\pi r^2h$

New Volume $=\pi(1.3r)^2(0.7h)$

$$=\frac{1183}{1000}\pi r^2h$$

Percentage change in Volume $=\dfrac{\frac{1183}{1000}\pi r^2h-\pi r^2h}{\pi r^2h}\times100\%$

$$=18.3\%$$

16 (a)

$$\frac{4}{2-3x}+\frac{4(x+7)}{(3x-2)(7x+3)}$$
$$=\frac{4(7x+3)-4(x+7)}{(2-3x)(7x+3)}$$
$$=\frac{28x+12-4x-28}{(2-3x)(7x+3)}$$
$$=\frac{24x-16}{(2-3x)(7x+3)} \text{ or } \frac{16-24x}{(3x-2)(7x+3)}$$
$$=-\frac{8}{(7x+3)}$$

(b)

$$\frac{3p}{q^2}\times\left(\frac{r^3}{3q^2}\right)^5$$
$$=3pq^{-2}\times r^{15}\times\frac{q^{-10}}{3^5}$$
$$=\frac{1}{81}pq^{-12}r^{15}$$
$$=\frac{pr^{15}}{81q^{12}}$$

(c)

$$\frac{m}{k}=\sqrt{\frac{3n-7}{4}}$$
$$\frac{m^2}{k^2}=\frac{3n-7}{4}$$
$$\frac{4m^2}{k^2}+7=3n$$
$$n=\frac{4m^2}{3k^2}+\frac{7}{3} \text{ or } n=\frac{4m^2+7k^2}{3k^2}$$

17 (a) $\frac{6x}{y^3}\times x^2y^4=6x^3y$ (b)(i) $(2^x)^3=5^3=125$ (b)(ii) $\frac{2}{2^x}=\frac{2}{5}$