Name:	Target Grade:	Actual Grade:

BRIGHT CULTURE

EXPONENTIAL AND LOGARITHMS

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Find a quiet, comfortable spot free place from distractions.

2. Spend one minute on each mark.

3. Time yourself for every single question.

4. Every chapter has their own question types. Ensure that you know the different question type for each chapter.

5. Make a conscientious effort to remember your mistakes, especially in terms of answering techniques. E.g Take a picture for the mistakes that you made, keep it in a photo album, and revise it over and over again.

6. Highlight question types that you tend to keep making mistakes and review them nearing exams.

7. Always review the common questions and question type that you tend to make mistakes nearing exams.

8. During exams, classify the question type and recall what you have learnt, how you need to analvse the questions for the different question type, what you need to take note of and answer with the correct answering techniques!

✨ Wishing you all the best for this test!

You've got this!

💡 With lots of love,
Bright Culture 💛

MARKS

If you are struggling in this paper, means you need to work harder!

If you need any professional guidance and further advice on how to advance, feel free to WhatsApp us at 91870820 or find us at [www.bright-culture.com/.](http://www.bright-culture.com/) We are committed to connect you to your future to reach your goals.

CHAPTER 4 EXPONENTIAL AND LOGARITHMS

1 **(a)** Solve the equation $\log_3(x-2)=\log_3(12-x)-2$. [4]

(b) Given that $(2\log_5 y)(\log_x 5)=8$, express y in terms of x. [3]

(c) Express $e^{4x} + 7 = 3e^{2x}$ as a quadratic equation in e^{2x} and explain why there are no real solutions. [3]

2 Given that $\frac{49^{2x-3}}{7^{3x}}$ - $\frac{7^{2x+1}}{343^{x-1}}$, find the value of $\sqrt[3]{343^x}$ [4]

3 Solve the equation

(i) $2\log_3(x+4)-\log_3(x+2)=2$, [5]

(ii) $\log_5 y-3\log_y 5=2$. [4]

4 The value of a newly mined diamond increases by half its value every 10 years. It is given that V_o is the value of the diamond at a particular time and V is its value t years later.

Find the value of the constant m in the relationship $V = V_o e^{mt}$. [3]

5 It is given that $\lg(a+2) = \log_{100}(b-1)$.

(i) Express b in terms of a. [3]

(ii) Given that $b \geq 10$, find the range of values for a. [3]

6 Given that $y = 3^x$, solve the equation $6(3^{-2x}) - 1 = 3^{-x}$. [4]

7 Find the values of x andy which satisfy the equations

$$4^{x+y} \times 2^{-3y} = 1,$$

$$\frac{81^x}{3^{3x+2y}} = \frac{1}{27}.$$

[4]

8 **(a)** Given that lg $z=k$, find an expression, in terms of k, for $\log_z(10z)$. [3]

(b) Solve the equation $\log_4(2-x)-1=\log_{16}(x+1)+\log_4\left(\frac{1}{16}\right)$. [5]

9 **(i)** Sketch the graph of $y = e^x$. [2]

(ii) By inserting an appropriate straight line on the same sketch in **(i),** state the number of solution(s) for the equation $\frac{1}{2}x = \ln\sqrt{3x+2}$. [2]

10 (a) Given that : $3^{n+2} - 3^{n} = \frac{5^{n+1}}{25^{n}}$, find the value of 15^{n}.

In an experimental environment, the population of a type of insect was observed. Over a period of 10 days from the start of the experiment, the number of insects decreased from 1100 to 600. The insect population is given by the formula $P = A + 900e^{kt}$, where A and k are constants and t is the number of days from the start of the experiment.

(a) Find the value of A and of k. [3]

(b) Explain why the population of the insects approaches the value of A after a long period of time. [1]

11 (a) Solve the equation $\log_2(x+2) - \log_{\sqrt{2}}(x-1) = 3$. [5]

(b) The curve $y = ax^b + 7$, where a and b are constants, passes through the points

$(2 , 47)$, $(-3 , -128)$ and $(5 , k)$. Find the values of a , b and k. [5]

12 The mass, m grams, of a radioactive substance, present at time t days after first being observed, is given by the formula $m = 30\,e^{-0.025t}$.

(i) Find the mass remaining after 30 days. [2]

(ii) Find the number of days required for the mass to drop to half of its initial value. Give your answer correct to the nearest integer. [2]

(iii) State the value m approaches when t becomes large. [1]

ANSWERS

1 (a) $x = 3$ (b) $y = x^4$

(c) $(e^{2x})^2 + 3e^{2x} + 7 = 0$.

Since discriminant < 0, there are no real solutions for the equation. (Or equivalent methods)

2 Given that $\frac{49^{2x-3}}{7^{3x}} = \frac{7^{2x+1}}{343^{x-1}}$, find the value of $\sqrt[3]{343^x}$.

$$\frac{49^{2x-3}}{7^{3x}} = \frac{7^{2x+1}}{343^{x-1}}$$

$$\frac{(7^2)^{2x-3}}{7^{3x}} = \frac{7^{2x+1}}{(7^3)^{x-1}}$$ express as powers of 7

$$\frac{7^{2(2x-3)}}{7^{3x}} = \frac{7^{2x+1}}{7^{3(x-1)}}$$ index laws $(a^m)^n = a^{mn}$

$$7^{4x-6-3x} = 7^{2x+1-(3x-3)}$$ $\frac{a^m}{a^n} = a^{m-n}$

$$x - 6 = -x + 4$$ equate powers

$$2x = 10$$

$$x = 5$$

$$343^{\frac{x}{3}} = 7^x$$

$$= 16807$$

3 Solve the equation

(i) $2\log_3(x+4)-\log_3(x+2)=2$, [5]

$$2\log_3(x+4)-\log_3(x+2)=2$$

$$\log_3(x+4)^2-\log_3(x+2)=2$$

$$\log_3\frac{(x+4)^2}{x+2}=2$$

$$\frac{(x+4)^2}{x+2}=3^2$$

$\log_a b^c = c\log_a b$

$\lg P - \lg Q = \lg\frac{P}{Q}$

$\log_a b = x \Longrightarrow a^x = b$

$$x^2+8x+16=9(x+2)$$

$$=9x+18$$

$$x^2-x-2=0$$

$$(x-2)(x+1)=0$$

$$x=2 \quad x=-1$$

(ii) $\log_5 y-3\log_y 5=2$. [4]

$$\log_5 y-3\log_y 5=2$$

$$\log_5 y-3\left(\frac{\log_5 5}{\log_5 y}\right)=2$$

$\log_a b = \frac{\log_c b}{\log_c a}$

$$(\log_5 y)^2-3=2\log_5 y$$

$(\log_5 y)^2 \neq 2(\log_5 y)$

$$(\log_5 y)^2-2\log_5 y-3=0$$

$$a^2-2a-3=0$$

$$(a+1)(a-3)=0$$

$$a=-1 \qquad a=3$$

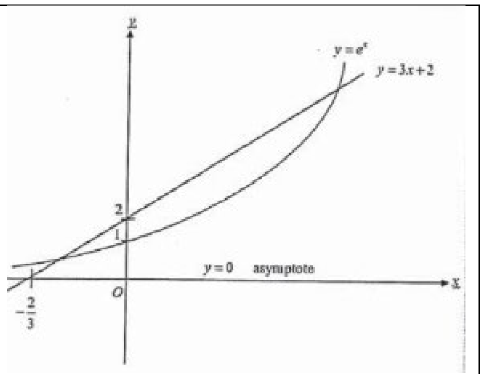
$$\log_5 y=-1 \quad \log_5 y=3$$

4	$m = 0.0405$
5i	$b = (a+2)^2 + 1$
5ii	a ;:: 1 or a :Σ –5 (reject)

6 $x = 0.631$

7 ***(b)***	*x=l, y='2*

8 (a)	$\frac{1}{k}+1$
(b)	1.60

9 (i)	
(ii)	Drawy=3x+2 From the graph, there are two solutions.

10 (a) Given that $3^{n+2} - 3^n = \dfrac{5^{n+1}}{25^n}$, find the value of 15^n.

$$3^n 3^2 - 3^n = \frac{5^{n+1}}{5^{2n}}$$

[M1]

$$3^n(3^2 - 1) = 5^{n+1-2n}$$

$$3^n(8) = 5^{1-n}$$

$$= 5^1 5^{-n} \quad \text{[M1]}$$

$$3^n(5^n) = \frac{5}{8}$$

$$15^n = \frac{5}{8} \quad \text{[A1]}$$

11 In an experimental environment, the population of a type of insect was observed. Over a period of 10 days from the start of the experiment, the number of insects decreased from 1100 to 600. The insect population is given by the formula $P = A + 900e^{kt}$, where A and k are constants and t is the number of days from the start of the experiment.

(a) Find the value of A and of k. [3]

$P = A + 900e^{kt}$

when $t = 0$, $P = 1100$,

$1100 = A + 900$

$A = 200$. [B1]

when $t = 10$, $P = 600$,

$600 = 200 + 900e^{10k}$

$e^{10k} = \frac{4}{9}$ [M1]

$10k = \ln\frac{4}{9}$

$k = -0.0811$ (to 3s.f.) [A1]

(b) Explain why the population of the insects approaches the value of A after a long period of time. [1]

As t increases, e^{kt} will approach 0, so P will approach 200. [B1]

12(a)	$\log_2(x+2) - \dfrac{\log_2(x-1)}{\log_2\sqrt{2}} = 3$ $\log_2(x+2) - 2\log_2(x-1) = 3$ $\log_2\dfrac{(x+2)}{(x-1)^2} = 3$ $\dfrac{x+2}{(x-1)^2} = 8$ $x+2 = 8(x-1)^2$ $8x^2 - 17x + 6 = 0$ $x = \dfrac{17 \pm \sqrt{(-17)^2 - 4(8)(6)}}{2(8)}$ $x = \dfrac{17 \pm \sqrt{97}}{16}$ $x = 1.68$, 0.447 (rejected)	M1 M1 M1 M1 A1 [deduct 1 m if did not reject 0.447]
12(b)	Sub $x = 2$, $y = 47$ $a \times 2^b + 7 = 47$ $a \times 2^b = 40$ ------------ (1) Sub $x = -3$, $y = -128$ $a \times (-3)^b + 7 = -128$ $a \times (-3)^b = -135$ ---------- (2) $\dfrac{(1)}{(2)}$ $\dfrac{2^b}{(-3)^b} = \dfrac{40}{-135}$ $\left(\dfrac{2}{-3}\right)^b = -\dfrac{8}{27} = \left(\dfrac{2}{-3}\right)^3$ $b = 3$ Sub $b = 3$ into (1) $a \times 2^3 = 40$ $a = 5$ $y = 5x^3 + 7$ Sub $x = 5$, $y = k$ $k = 5(5)^3 + 7 = 632$	M1 for eqn (1) & (2) M1 A1 A1 A1

13	The mass, m grams, of a radioactive substance, present at time t days after first being observed, is given by the formula $m = 30\,e^{-0.025t}$. **(i)** Find the mass remaining after 30 days. [2] **(ii)** Find the number of days required for the mass to drop to half of its value at $t = 0$. Give your answer correct to the nearest integer. [2] **(iii)** State the value m approaches when t becomes large. [1]	
(i)	$m = 30\,e^{-0.025(30)}$ $= 14.171$ $= 14.2$ The remaining mass after 30 days is 14.2g.	M1 A1 – 0 mark for omission of unit in answer
(ii)	$15 = 30\,e^{-0.025t}$ $e^{-0.025t} = \frac{1}{2}$ $-0.025t = \ln\frac{1}{2}$ $t = 27.726$ $t = 28$ The number of days required is 28 days.	B1 A1
(iii)	As $t \longrightarrow \infty$, $30\,e^{-0.025t} \longrightarrow \infty$, the value of m approaches 0.	A1