Name:	Target Grade:	Actual Grade:

BRIGHT CULTURE

BINOMIAL THEOREM

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Find a quiet, comfortable spot free place from distractions.
2. Spend one minute on each mark.
3. Time yourself for every single question.
4. Every chapter has their own question types. Ensure that you know the different question type for each chapter.
5. Make a conscientious effort to remember your mistakes, especially in terms of answering techniques. E.g Take a picture for the mistakes that you made, keep it in a photo album, and revise it over and over again.
6. Highlight question types that you tend to keep making mistakes and review them nearing exams.
7. Always review the common questions and question type that you tend to make mistakes nearing exams.
8. During exams, classify the question type and recall what you have learnt, how you need to analvse the questions for the different question type, what you need to take note of and answer with the correct answering techniques!

✨ Wishing you all the best for this test!

You've got this!

💡 With lots of love,
Bright Culture 💛

MARKS

If you are struggling in this paper, means you need to work harder!

If you need any professional guidance and further advice on how to advance, feel free to WhatsApp us at 91870820 or find us at www.bright-culture.com/. We are committed to connect you to your future to reach your goals.

CHAPTER 5 BINOMIAL THEOREM

1 **(i)** Write down, and simplify, the first 4 terms in the expansion $(1+x)^7$ in ascending powers of x. [2]

(ii) Write down the general term in the binomial expansion of $\left(x^2-\frac{2}{x^3}\right)^9$. [1]

(iii) Write down the power of x in this general term. [1]

(iv) Hence, or otherwise, determine the coefficient of x^3 in the expansion of

$(1+x)^7 + \left(x^2-\frac{2}{x^3}\right)^9$.

2 **(i)** Write down, and simplify, the general term in the binomial expansion of $\left(x^2+\frac{3}{x}\right)^{15}$. [2]

(ii) Hence determine the coefficient of x^3 in the binomial expansion of $\left(x^2+\frac{3}{x}\right)^{15}$. [3]

(iii) Explain why there is no term in $\frac{1}{x^5}$ in the expansion of $\left(x^2+\frac{3}{x}\right)^{15}$. [2]

3 The third term in the binomial expansion of $\left(x - \frac{3}{x}\right)^n$, where n is a positive integer, is kx^8 .

(i) 12/3/2024Show that $n = 12$. [2]

(ii) Find the value of k. [1]

(iii) Find the coefficient of x^8 in the expansion of $\left(x - \frac{3}{x}\right)^{12}\left(2 + x^2\right)$.

4 **(a)** Find the term independent of x in the expansion of $\left(x-\dfrac{1}{2x^3}\right)^{16}$. [3]

(i) **(i)** Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(1-\frac{x}{3}\right)^n$, where n is a positive integer. [2]

(ii) The first three terms in the expansion of $(m + x - x^2)\left(1-\frac{x}{3}\right)^n$, are $4 - 7x + kx^2$.
Find the value of each of the constants m, n and k.

[5]

5 **(a)** The sum of the coefficients of the first two terms in the expansion, in descending powers of x, of $(1 + 2x)\left(2x - \frac{1}{x^2}\right)^n$ is 768, where n is a positive integer greater than 2. Show that n is 8

[4]

(b) Find the term containing x^{-7} in the expansion of $\left(2x - \frac{1}{x^2}\right)^8$.

6 **(a)** Find, in ascending powers of x , the first three terms in the expansion of $(2-x)^7$. [2]

Hence, find the value of the constant a for which the coefficient of x^2 in the expansion of $(a-x)(2-x)^7$ is 616. [3]

(b) In the expansion of $\left(x^2-\frac{1}{2x^4}\right)^n$ in descending powers of x, the sixth term is independent of x . Find the value of n and the term independent of x. [4]

ANSWERS

1 Try it yourself (:

2 **(i)** Write down, and simplify, the general term in the binomial expansion of $\left(x^2+\frac{3}{x}\right)^{15}$. [2]

(i) $\left(x^2+\frac{3}{x}\right)^{15}$

$$T_{r+1} = {}^{15}C_r\left(x^2\right)^{15-r}\left(\frac{3}{x}\right)^r$$

$$= {}^{15}C_r x^{30-2r}\left(\frac{3^r}{x^r}\right)$$

$$= {}^{15}C_r 3^r x^{30-3r}$$

(ii) Hence determine the coefficient of x^3 in the binomial expansion of $\left(x^2+\frac{3}{x}\right)^{15}$. [3]

(ii) $30-3r=3$

$3r=27$

$r=9$

Coefficient of $x^3 = {}^{15}C_9 3^9$

$= (5005)(19683)$

$= 98\ 513\ 415$

(iii) Explain why there is no term in $\frac{1}{x^5}$ in the expansion of $\left(x^2+\frac{3}{x}\right)^{15}$.

(iii) $30-3r=-5$

$3r=35$

$r=\frac{35}{3}$

r must be a whole number. $\therefore$ no term in $\frac{1}{x^5}$

3 (ii) 594 (iii) −4752

4 (a)	$\frac{455}{4}$
4 (b) (i)	$1-\frac{n}{3}x+\frac{n(n-1)}{18}$
4 (b) (ii)	$m = 4$, $n = 6$, $k = \frac{11}{3}$

5 (a) The sum of the coefficients of the first two terms in the expansion, in descending powers of x, of $(1+2x)\left(2x-\frac{1}{x^2}\right)^n$ is 768, where n is a positive integer greater than 2. Show that n is 8

[4]

$$\left(2x-\frac{1}{x^2}\right)^n = \binom{n}{0}(2x)^n + \binom{n}{1}(2x)^{n-1}\left(-\frac{1}{x^2}\right) + \dots \quad [M1]$$

$$= 2^n x^n - n(2^{n-1})(x^{n-1})(x^{-2}) + \dots$$

$$= 2^n x^n - n(2^{n-1})(x^{n-3}) + \dots$$

$$(1+2x)\left(2x-\frac{1}{x^2}\right)^n = (1+2x)(2^n x^n - n2^{n-1}x^{n-3} + \dots)$$

$$= \underbrace{2^n x^n}_{T_2} - n2^{n-1}x^{n-3} + \underbrace{2^{n+1}x^{n+1}}_{T_1} - n2^n x^{n-2} + \dots \quad [M1]$$

$$2^{n+1} + 2^n = 768 \quad [M1]$$

$$2^n(2+1) = 768$$

$$2^n = 256 \quad [A1]$$

$n = 8$ (shown)

(b) Find the term containing x^{-7} in the expansion of $\left(2x-\frac{1}{x^2}\right)^8$.

$$\left(2x-\frac{1}{x^2}\right)^8$$

$$T_{r+1} = \binom{8}{r}(2x)^{8-r}\left(-\frac{1}{x^2}\right)^r \quad [M1]$$

$$= \binom{8}{r}(2)^{8-r}(x)^{8-r}(-1)^r(x)^{-2r}$$

$$= \binom{8}{r}(2)^{8-r}(-1)^r(x)^{8-3r} \quad [M1]$$

$$8-3r=-7$$

$$3r=15$$

$$r=5 \quad [M1]$$

$$T_6 = \binom{8}{5}(2)^{8-5}(-1)^5(x^{-7})$$

$$= -448x^{-7} \quad [A1]$$

6	**(a)** Find, in ascending powers of x, the first three terms in the expansion of $(2-x)^7$. [2] Hence, find the value of the constant a for which the coefficient of x^2 in the expansion of $(a-x)(2-x)^7$ is 616. [3] **(b)** In the expansion of $\left(x^2-\frac{1}{2x^4}\right)^n$ in descending powers of x, the sixth term is independent of x. Find the value of n and the term independent of x. [4]	
(a)(i) (ii)	$(2-x)^7 = 2^7 - \binom{7}{1}(2^6)x + \binom{7}{2}(2^5)x^2 + \ldots$ $= 128 - 448x + 672x^2 + \ldots$ $(a-x)(2-x)^7$ $= (a-x)(128 - 448x + 672x^2 + \ldots)$ $= \ldots + 672ax^2 + 448x^2 + \ldots$ coefficient of x^2: $672a + 448 = 616$ $a = \frac{1}{4}$	M1 A1 M1 M1 A1
(b)	$\left(x^2-\frac{1}{2x^4}\right)^n$ $T_6 = \binom{n}{5}\left(x^2\right)^{n-5}\left(-\frac{1}{2x^4}\right)^5$ $= \binom{n}{5}\left(x^{2n-10}\right)\left(-\frac{1}{2}\right)^5 x^{-20}$ $= \binom{n}{5}\left(x^{2n-30}\right)\left(-\frac{1}{2}\right)^5$ **As it is independent of x** **$2n - 30 = 0$** $n = 15$ Value of the term $= \binom{15}{5}\left(-\frac{1}{2}\right)^5$ $= -\frac{3003}{32}$ or $-93\frac{27}{32}$	M1 A1 M1 A1