Name:	Target Grade:	Actual Grade:

BRIGHT CULTURE

TRIGONOMETRY

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Find a quiet, comfortable spot free place from distractions.
2. Spend one minute on each mark.
3. Time yourself for every single question.
4. Every chapter has their own question types. Ensure that you know the different question type for each chapter.
5. Make a conscientious effort to remember your mistakes, especially in terms of answering techniques. E.g Take a picture for the mistakes that you made, keep it in a photo album, and revise it over and over again.
6. Highlight question types that you tend to keep making mistakes and review them nearing exams.
7. Always review the common questions and question type that you tend to make mistakes nearing exams.
8. During exams, classify the question type and recall what you have learnt, how you need to analyse the questions for the different question type, what you need to take note of and answer with the correct answering techniques!

✨ Wishing you all the best for this test!

You've got this!

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MARKS

If you are struggling in this paper, means you need to work harder!

If you need any professional guidance and further advice on how to advance, feel free to WhatsApp us at 91870820 or find us at www.bright-culture.com/. We are committed to connect you to your future to reach your goals.

CHAPTER 8: TRIGONOMETRY

1 It is given that $6\sin^2 A - 4\cos^2 A = 5\sin 2A$ where $0 \le A \le 90°$.

(i) Show that $\tan A = 2$. [3]

(ii) **Hence,** find the value of $\cos\left(60^\circ + A\right)$, leaving your answer in the form $\dfrac{a + b\sqrt{3}}{2\sqrt{5}}$. [3]

(iii) Without finding the value of A, explain whether $60° + A$ is acute or obtuse [1]

2 **(i)** Show that $\frac{cosec8+sin8}{cosec8-sin8} = 2\sec^2 8 - 1$. [4]

(ii) Hence, solve the equation $\dfrac{\text{cosec}2x + \sin 2x}{\text{cosec}2x - \sin 2x} = 3$ for $0 \le x \le \dfrac{\pi}{2}$, leaving your answers in terms of π. [4]

3 It is given that $f(x) = 3\cos\frac{x}{2} + K$, where k is a constant. The graph of $y = f(x)$ passes through the point $(\pi, -1)$.

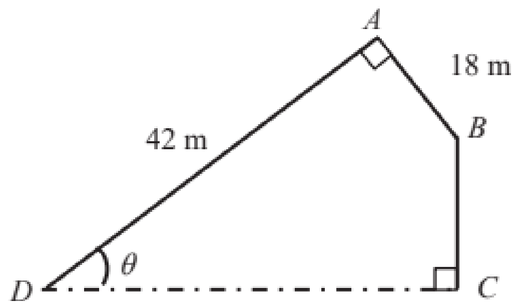
(i) State the amplitude and period of $f(x)$. [2]

(ii) Find the value of k. [2]

(iii) Sketch the graph of $y = f(x)$ for $0 \le x \le 3\pi$. [3]

(iv) By drawing a suitable straight line to the sketch in **(iii)**, find the number of solutions to the equation $f(x) + 4 = \frac{x}{2\pi}$ for $0 \le x \le 3\pi$. [1]

4



The diagram shows a plot of land being fenced up along AB, BC and AD, where $AB = 18$ m, $AD = 42$ m, angle DAB = angle $BCD = 90^\circ$ and the acute angle $ADC = \theta$ can vary.

(i) Show that L m, the length of the fence, can be expressed as $L = 60 + 42\sin\theta - 18\cos\theta$. [2]

(ii) Express L in the form $p + R\sin(\theta - \alpha)$, where p and $R > 0$ are constants and $0° < \alpha < 90°$. [4]

(iii) Given that the exact length of the fence used is 92 m, find the value of θ. [3]

5 (i) Without using a calculator, show that $\cot 15° = 2 + \sqrt{3}$. [4]

(ii) Hence show that $\operatorname{cosec}^2 15° = 4\cot 15°$. [2]

6 It is given that $y_1 = \sin x - 2$ and $y_2 = -3\cos 2x$.

(i) State the amplitude and the period, in degrees, of **(a)** y_1 , **(b)** y_2 [2]

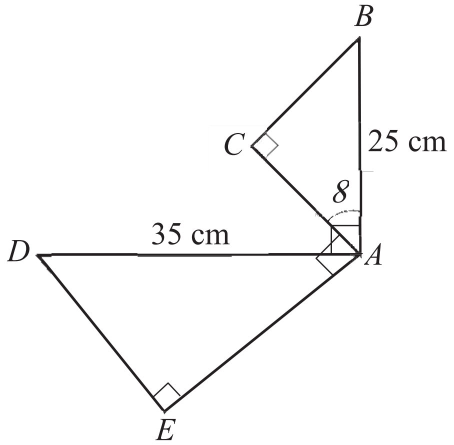
For the interval $0° \leq x \leq 360°$,

(ii) solve the equation $y_1 = y_2$, [4]

(iii) sketch, on the same diagram, the graphs of y_1 and y_2 , [4]

(iv) find the set of values of x for which $y_2 - y_1 > 0$. [2]

7



In the diagram, A, B and D are fixed points such that $AB = 25$cm, $AD = 35$ cm and angle $BAD = 90°$. Angle $BAC = \theta$ and can vary. AC is perpendicular to BC, EA is perpendicular to AC, and DE is perpendicular to EA.

(i) Show that $AC + BC + DE = (60\sin\theta + 25\cos\theta)$ cm. [2]

(ii) Express $AC + BC + DE$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) **Without evaluating** θ, explain why $AC + BC + DE$ cannot have a length of 70 cm. [1]

(iv) Find the value of θ for which $AC + BC + DE = 50$ cm [2]

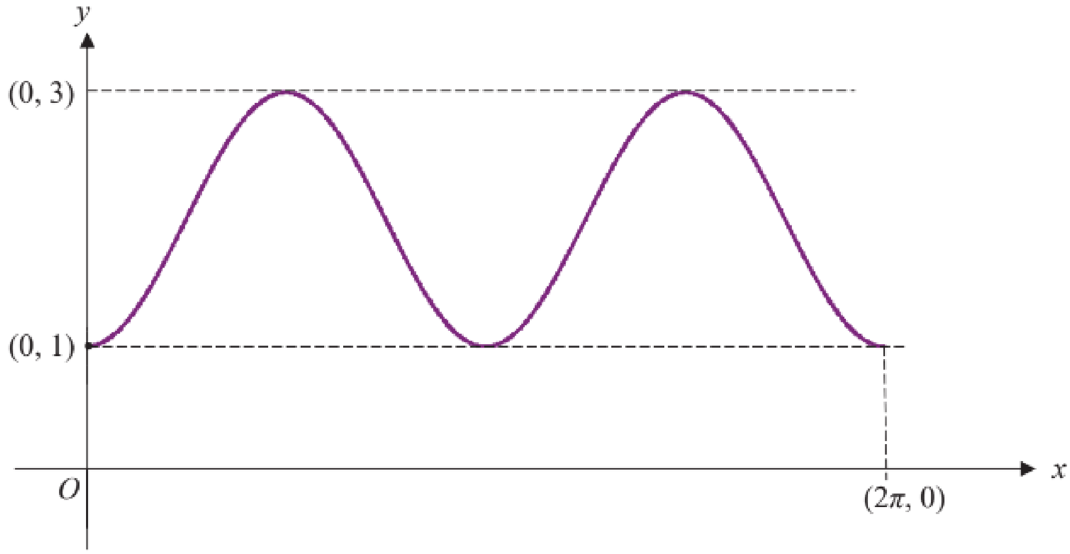
8 It is given that $\cos A = -m$, where $m > 0$, and that A is obtuse.
Find the value of each of the following in terms of m.

(a) $\tan A$ [2]

(b) $\cot(180 - A)$ [1]

(c) $\cos\left(\frac{A}{2}\right)$ [3]

9



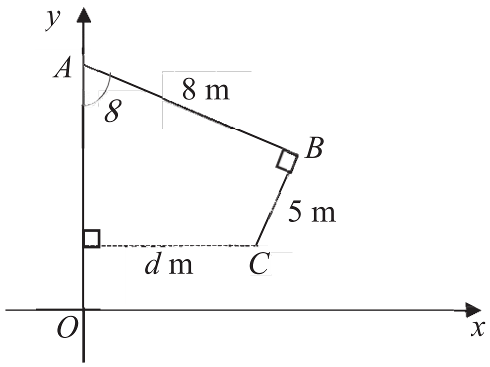
The diagram shows the curve $y = a + b\cos(cx)$ for $0 \le x \le 2\pi$.

(i) Write down the value of a, of b and of c. [3]

(ii) Sketch, on the same diagram, the graph of $y = \sin\left(\frac{x}{2}\right) + 2$ for $0 \le x \le 2\pi$. [3]

(iii) Deduce the largest integer value of k such that $a+b\cos(cx) > \sin\left(\frac{x}{2}\right)+k$ for $0 \le x \le 2\pi$. [1]

10 The diagram shows a rod AB which is hinged at A, and a rod BC which is fixed at B such that angle $ABC = 90°$. The rods can move in the xy-plane with origin O where the x and y axes are horizontal and vertical respectively. The rod AB can turn about A and is inclined at an angle 8 to the y-axis, where 0° ⎬ 8 ⎬ 180°. The lengths of AB and BC are 8 m and 5 m respectively.



Given that C is d m from the y-axis,

(i) find the values of a and b for which $d = a \sin 8 - b \cos 8$ [2]

Using the values of a and b found in part (i),

(ii) express d in the form $R \sin (8 - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

Hence

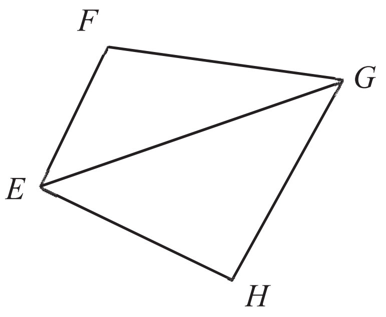
(iii) explain if it is possible for d to be 10 m, [2]

(iv) find the value(s) of θ when $d = 6$ m. [2]

11 (i) Prove that $\dfrac{\operatorname{cosec}^2\theta - 2}{\operatorname{cosec}^2\theta} = \cos 2\theta$. [3]

(ii) Hence solve the equation $\dfrac{\operatorname{cosec}^2\theta - 2}{\operatorname{cosec}^2\theta} + \dfrac{3}{\operatorname{cosec} 2\theta} = 0$ for $0 < \theta < 5$. [4]

12



EFGH is a plot of land that comprises two smaller plots, triangle *EFG* and triangle *EGH*.

EFGH is a plot that comprises two smaller plots, triangle *EFG* and triangle *EGH*.
EF and *EH* are perpendicular, angle $FEG = \theta$, $EH = 42$ m, $EG = 55$ m and $EF = 48$ m.

(i) Show that the area, A m^2, of *EFGH* can be expressed as
$A = 1320\sin\theta + 1155\cos\theta$. [2]

(ii) Express A in the form in the form $R\sin(\theta + \alpha)$, where

$R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the value of θ if the area is 1231 m^2. [2]

13 Solve the equation $3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)$, giving the principal values of x, in radians. [5]

14 (a) (i) Prove the identity $\sin x \cos x + \cot x \cos^2 x = \cot x$. [4]

(ii) Hence, solve $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1$ for $0 \le x \le \pi$. [3]

(b) **(i)** On the same axes, sketch the graphs of $y = 3\sin x - 1$ and $y = \tan\frac{x}{2}$ for $0 \le x \le 2\pi$.

[5]

(ii) Hence, state the number of solutions of $3\sin x - 1 = \tan\frac{x}{2}$ for $0 \le x \le 2\pi$.

[1]

15 Given that $\cos A = p$ and that A is acute, express the following in terms of p.

(i) $\sin 2A$ [3]

(ii) $\cos \frac{A}{2}$ [3]

16 Given that θ is obtuse and $\tan\theta = a$, express, in terms of a,

a. $\cos\theta$, [3]

b. $\operatorname{cosec}\theta$. [2]

17 It is given that $\mathbf{f(x) = 2\sin\frac{x}{2}}$ and $g(x) = 3\cos x + 1$ where $0 \le x \le 2\pi$.

(i) State the period of $f(x)$. [1]

(ii) State the smallest value of $f(x)$. [1]

(iii) State the largest value of $g(x)$. [1]

(iv) State the largest value of $|f(x) - g(x)|$. [1]

(v) Sketch, on the same axes, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ for $0 \le x \le 2\pi$. [4]

(vi) Given that the solutions to the equation $\mathrm{f}(x) = \mathrm{g}(x)$ for $0 \le x \le 2\pi$ are a and b where $a < b$, state the range of value of x for which $\mathrm{f}(x) \ge \mathrm{g}(x)$. [1]

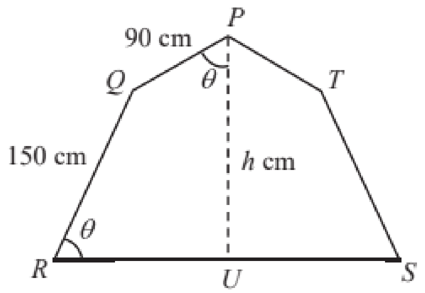
18 **(a)** It is given that $\tan(A + B) = 8$ and $\tan B = 2$. **Without using a calculator**, find the exact value of $\cot A$. [3]

(b) **(i)** Prove that $\sin 2x\,(\cot x - \tan x) = 2\cos 2x$. [3]

(ii) Hence solve the equation $\sin 2x(\cot x - \tan x) = \sec 2x$ for $0 \le x \le \pi$.

[4]

19 The diagram shows the side view *PQRST* of a tent. The tent rests with *RS* on horizontal ground. *PQRST* is symmetrical about the vertical *PU*, where *U* is the midpoint of *RS*. Angle QPU = angle QRU = θ radians and the lengths of *PQ* and *QR* are 90 cm and 150 cm respectively. The vertical height of *P* from the ground is *h* cm.



(i) Explain clearly why $h = 90\cos\theta + 150\sin\theta$. [2]

(ii) Express *h* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Find the greatest possible value of h and the value of θ at which this occurs. [3]

(iv) Find the values of θ when $h = 160$. [3]

20 (i) Show that $\dfrac{\tan^2 x - 1}{\tan^2 x + 1} = 1 - 2\cos^2 x$. [3]

(ii) Hence find, for $0 \le x \le 5$, the values of x in radians for which $\dfrac{\tan^2 x - 1}{\tan^2 x + 1} = \dfrac{1}{2}$. [3]

ANSWERS

1 Given that $6\sin^2 A - 4\cos^2 A = 5\sin 2A$ where $0 \geq A \geq 90°$.

(i) Show that $\tan A = 2$. [3]

$6\sin^2 A - 4\cos^2 A = 5\sin 2A$

$6\sin^2 A - 4\cos^2 A = 5(2\sin A\cos A)$

$3\sin^2 A - 5\sin A\cos A - 2\cos^2 A = 0$

$(3\sin A + \cos A)(\sin A - 2\cos A) = 0$

$3\sin A = -\cos A$ or $\sin A = 2\cos A$

$\frac{\sin A}{\cos A} = -\frac{1}{3}$ or $\frac{\sin A}{\cos A} = 2$

$\tan A = -\frac{1}{3}$ (rej. ∵ A is acute) or $\tan A = 2$ (shown)

Alternative Method

$6\sin^2 A - 4\cos^2 A = 5\sin 2A$

$6\sin^2 A - 4\cos^2 A = 5(2\sin A\cos A)$

$\frac{6\sin^2 A}{\cos^2 A} - \frac{4\cos^2 A}{\cos^2 A} = \frac{10\sin A\cos A}{\cos^2 A}$

$6\tan^2 A - 4 = 10\tan A$

$6\tan^2 A - 10\tan A - 4 = 0$

$3\tan^2 A - 5\tan A - 2 = 0$

$(3\tan A + 1)(\tan A - 2) = 0$

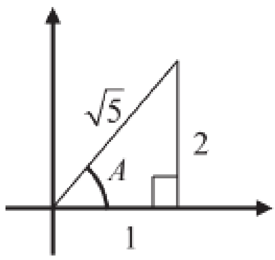
$\tan A = 2$ (shown) or $\tan A = -\frac{1}{3}$ (rej.)

(ii) **Hence,** find value of $\cos(60° + A)$, leaving your answer in the form $\frac{a + b\sqrt{3}}{2\sqrt{5}}$

$\cos(60° + A) = \cos 60° \cos A - \sin 60° \sin A$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{5}}\right) - \frac{\sqrt{3}}{2}\left(\frac{2}{\sqrt{5}}\right)$

$= \frac{1 - 2\sqrt{3}}{2\sqrt{5}}$



[3]

(Draw a diagram to find the other trigo. Ratios in terms of A)

(iii) Without finding the value of A , explain whether $60° + A$ is acute or obtuse. [1]

Since $1 - 2\sqrt{3}$ is negative, from (ii), $\cos(60° + A)$ is a negative ratio. So $60° + A$ lies either in the 2nd or 4th quadrant. Since both 60° and A are both acute, the addition of 2 acute angles cannot exceed 180° . Thus $60° + A$ lies in the 2nd quadrant and is obtuse.

2 **(i)** Show that $\dfrac{\operatorname{cosec}\theta+\sin\theta}{\operatorname{cosec}\theta-\sin\theta}=2\sec^2\theta-1$. [4]

(i) $$\frac{\operatorname{cosec}\theta+\sin\theta}{\operatorname{cosec}\theta-\sin\theta}=\frac{\frac{1}{\sin\theta}+\sin\theta}{\frac{1}{\sin\theta}-\sin\theta}$$

$$=\frac{\frac{1+\sin^2\theta}{\sin\theta}}{\frac{1-\sin^2\theta}{\sin\theta}}$$

$$=\frac{1+\sin^2\theta}{1-\sin^2\theta}$$

$$=\frac{1+\sin^2\theta}{\cos^2\theta}$$

$$=\frac{1+1-\cos^2\theta}{\cos^2\theta} \quad \text{OR} \quad =\frac{1}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}$$

$$=\frac{2}{\cos^2\theta}-\frac{\cos^2\theta}{\cos^2\theta} \quad \text{OR} \quad =\frac{1}{\cos^2\theta}+\tan^2\theta$$

$$=2\sec^2\theta-1 \qquad\qquad =\sec^2\theta+\sec^2\theta-1$$

$$=2\sec^2\theta-1$$

Comments

- When given reciprocal functions like cosec8, sec8 or cot8, it is advised to express it in terms of sin8, cos8 or tan8 .
- Students should then make attempts to simplify the fraction further before determining which identity would be useful to apply towards getting 2 $\sec^2 8-1$.
- It is also useful to work with the end in mind, thinking about what function would lead to$\sec^2 8$.

(ii) Hence, solve the equation $\dfrac{\operatorname{cosec}2x+\sin 2x}{\operatorname{cosec}2x-\sin 2x}=3$ for $0\le x\le\dfrac{\pi}{2}$, leaving your answers in terms of π. [4]

(ii) $$\frac{\operatorname{cosec}2x+\sin 2x}{\operatorname{cosec}2x-\sin 2x}=3$$

$$2\sec^2 2x-1=3$$

$$\sec^2 2x=2$$

$$\sec 2x=\pm\sqrt{2}$$

$$\cos 2x=\pm\frac{1}{\sqrt{2}}$$

$$\text{Basic angle}=\cos^{-1}\frac{1}{\sqrt{2}}=\frac{\pi}{4}$$

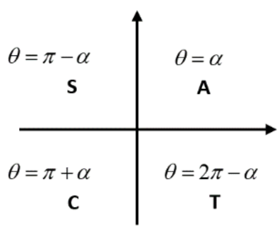
$$\text{New domain: } 0\le x\le\frac{\pi}{2}\Rightarrow 0\le 2x\le\pi$$

$$2x=\frac{\pi}{4}\ \text{or}\ \frac{3\pi}{4}$$

$$x=\frac{\pi}{8}\ \text{or}\ \frac{3\pi}{8}$$

Comments

- Most students were successful in seeing the relationship between (i) and (ii).
- A common mistake is to only give the positive square root and miss out the **negative square root**. It must be noted that when taking **square roots** to finding an unknown, the square roots could be *positive* or *negative*.
- Students should make it a point to consider the new domain (for 2x) so as not to miss out any possible angle that lies in the required quadrant.



3 It is given that $f(x) = 3\cos\frac{x}{2} + k$, where k is a constant. The graph of $y = f(x)$ passes through the point $(\pi, -1)$.

(i) State the amplitude and period of $f(x)$. [2]

(i) Amplitude = 3

Period $= \dfrac{2\pi}{\frac{1}{2}}$

$= 4\pi$

(ii) Find the value of k. [2]

(ii) At $(\pi, -1)$

$-1 = 3\cos\frac{\pi}{2} + k$

$-1 = 0 + k$

$k = -1$

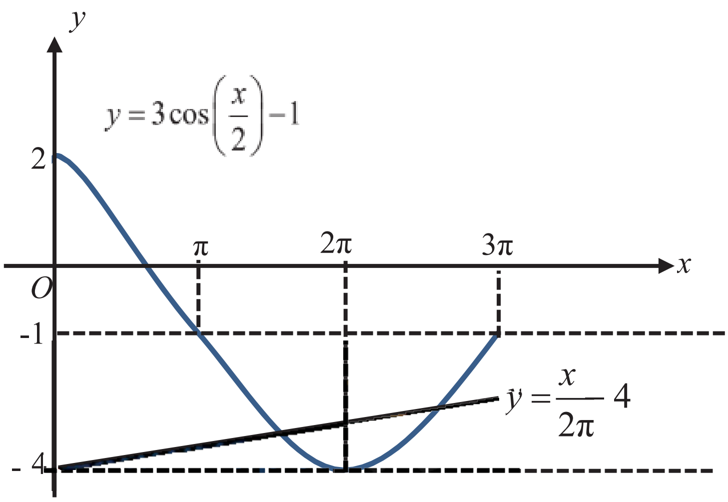
Comments

Students should show clear working when trying to find the value of k.

Evidence of substituting $(\pi, -1)$ into the equation should be seen in an attempt to find k.

(iii) Sketch the graph of $y = f(x)$ for $0 \le x \le 3\pi$. [3]

(iii)



Comments

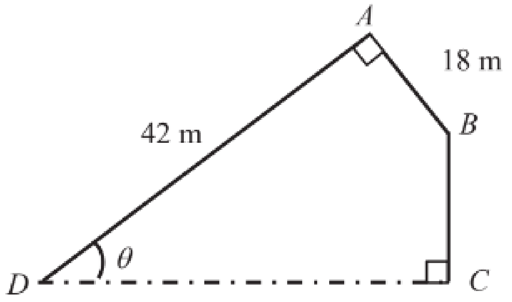
- Note that the graph must be sketched only for $0 \le x \le 3\pi$.
- The critical points of the graph (max/min point and points on the axis of the curve) must be indicated.

(iv) By drawing a suitable straight line to the sketch in **(iii)**, find the number of solutions to the equation $f(x) + 4 = \frac{x}{2\pi}$ for $0 \le x \le 3\pi$. [1]

Draw line $y = \frac{x}{2\pi} - 4$

Number of solutions = 2

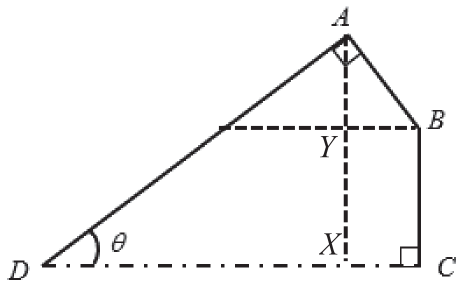
4



The diagram shows a plot of land being fenced up along *AB*, *BC* and *AD*, where *AB* = 18 m, *AD* = 42 m, angle *DAB* = angle *BCD* = 90° and the acute angle *ADC* = ϑ can vary.

(i) Show that *L* m, the length of the fence, can be expressed as
$L = 60 + 42\sin\vartheta - 18\cos\vartheta$. [2]

(i)



$\Lambda DAX = 90° - \theta$

$\Lambda XAB = \theta$

$AX = 42\ \sin\theta$

$BC = 42\ \sin\theta - 18\cos\theta$

$L = AD + AB + BC$

$= 42 + 18 + 42\ \sin\theta - 18\cos\theta$

$= 60 + 42\ \sin\theta - 18\cos\theta$

Comments

- Some attempts to "cut the angle *θ*" were observed. Since the final expression to be proved involves angle *θ*, it wouldn't be helpful to cut the angle up in any way.
- Instead attempts should be made to cut the diagrams up to get right angle triangles where the sides are related to the length from *A* to *DC*.
- Working must be shown clearly to show the components of the length of fence that result in $L = 60 + 42\ \sin\theta - 18\cos\theta$.

(ii) Express *L* in the form $p + R\sin(\theta - \alpha)$, where *p* and $R > 0$ are constants

and $0° < \alpha < 90°$. [4]

(ii) $42\sin\theta - 18\cos\theta = R\sin(\theta - \alpha)$

$= R\left(\sin\theta\cos\alpha - \cos\theta\sin\ \alpha\right)$

Comparing coefficients:

$R\ \sin\ \alpha = 18$ and $R\ \cos\alpha = 42$

$\tan\ \alpha = \dfrac{18}{42}$

$\alpha = 23.1985°$

$R = \sqrt{18^2 + 42^2} = \sqrt{2088} = 6\sqrt{58}$

$L = 60 + 6\sqrt{58}\sin(\theta - 23.2°)$

Comments

- The working on comparing the coefficients is required as evidence of understanding how *R* and α are related to the coefficients.
- Always provide greater accuracy to the intermediate values obtained for *R* and α , where applicable.

  These values are required in the subsequent parts to obtain more accurate results/answers.

(iii) Given that the exact length of the fence used is 92 m, find the value of θ. [3]

(iii)

$60+6\sqrt{58}\sin(\theta-23.1985°)=92$

$6\sqrt{58}\sin(\theta-23.1985°)=32$

$\sin(\theta-23.1985°)=\dfrac{32}{6\sqrt{58}}$

$\text{Basic angle}=\sin^{-1}\left(\dfrac{32}{6\sqrt{58}}\right)=44.4511°$

$\theta-23.1985°=44.4511°$

$\theta=44.4511°+23.1985°$

$\theta=67.6°$ (to 1 d.p.)

<u>Comments</u>

Check that the angle θ should be acute based on the context.

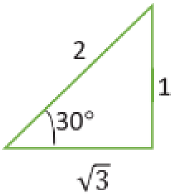
Hence, students should always make it a point to check that the answer is reasonable / makes sense in relation to the context.

5 **(i) Without using a calculator** , show that $\cot 15° = 2 + \sqrt{3}$. [4]

tan15°   tan(45° – 0°)   or tan15° tan (60° – 45°)

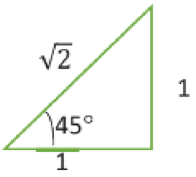
$$tan15° = \frac{\tan 45° - tan30°}{1 + tan45° tan30°} \text{ or } tan15° = \frac{tan60° - tan45°}{1 + tan45° tan60°}$$

$$tan15° = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} \qquad \text{or} \qquad \tan 15° = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$



$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$Cot15° = \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \qquad \cot A = \frac{1}{tanA}$$



$$= \frac{3 + 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 + 2\sqrt{3}}{2}$$

$$= \frac{2(2 + \sqrt{3})}{2} = 2 + \sqrt{3}$$

(ii) Hence show that $\operatorname{cosec}^2 15° = 4\cot 15°$. [2]

$$cosec^2 15° = 1 + \cot^2 15° = 1 + (2 + \sqrt{3})^2$$

$$= 1 + 4 + 4\sqrt{3} + 3$$

$$= 8 + 4\sqrt{3}$$

$$= 4(2 + \sqrt{3})$$

$$= 4cot15°$$

6 It is given that $y_1 = \sin x - 2$ and $y_2 = -3\cos 2x$.

(i) State the amplitude and period, in degrees, of **(a)** y_1 , **(b)** y_2 . [2]

(a) $y_1 = \sin x - 2$ amplitude =1, period=360°

(b) $y_2 = -3\cos 2x$ amplitude =3, period =180°

For the interval $0° \le x \le 360°$,

(ii) solve the equation $y_1 = y_2$, [4]

y1= y2

$\sin x - 2 = -\cos 2x$

$3(1 - 2\sin^2 x) + \sin x - 2 = 0$ cos2x 1 - 2 sin²x

$6\sin^2 x - \sin x - 1 = 0$

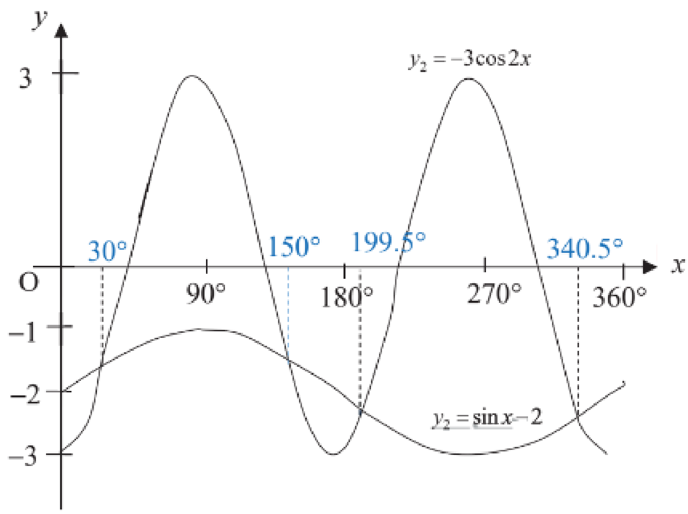
$(3\sin x\ 1)(2\sin x - 1) = 0$

$\sin x = -\frac{1}{3},$ $\sin x = \frac{1}{2}$

$x = 180° + 19.471°, 360° - 19.471,$ $x = 30°, 150°$

$x =$ 199.5°, 340.5°. x= 30°, 150°

(iii) sketch, on the same diagram, the graphs of y_1 and y_2 , [4]

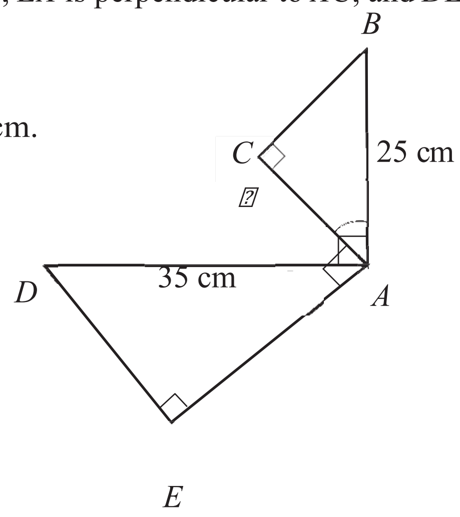


(iv) find the set of values of x for which $y_2 - y_1 > 0$. [2]

$0° < x < 150°, 199.5° < x < 40.5°$

7 In the diagram, A, B and D are fixed points such $AB = 25$ cm, $AD = 35$ cm and angle $BAD = 90°$.

Angle $BAC = \theta$ and can vary. AC is perpendicular to BC, EA is perpendicular to AC, and DE is perpendicular to EA.



(i) Show that $AC + BC + DE = (60\sin\theta + 25\cos\theta)$ cm. [2]

$BC = 25\sin\theta$, $AC = 25\cos\theta$,

$\angle CAD = 90° - \theta$, $\angle DAE = \theta$, $DE = 35\sin\theta$

$AC + BC + DE = 25\cos\theta + 25\sin\theta + 35\sin\theta$

$= 25\cos\theta + 60\sin\theta$

(ii) Express $AC + BC + DE$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

Let $60\sin\theta + 25\cos\theta = R\sin(\theta + a)$

$60\sin 0 + 25\cos\theta = R\sin\theta\cos a + R\cos\theta\sin a$

$R\cos a = 60$ ----(1) $R\sin\alpha = 25$ ----- (2)

$\frac{(2)}{(1)}$ $\frac{R\sin a}{R\cos a} = \frac{25}{60}$

$\tan\alpha = \frac{5}{12}$

$\alpha = 22.619°$

$R^2(\sin^2\alpha + \cos^2\alpha) = 60^2 + 25^2$

$R = \pm\sqrt{60^2 + 25^2}$

$R = 65$, $R = -65$ (NA)

$60\ \text{in}\theta + 25\cos\theta = 65\sin(\theta + 22.6°)$

(iii) Without evaluating θ, explain why $AC + BC + DE$ cannot have a length of 70 cm. [1]

$-1 \leq \sin(\theta + 22.6°) \leq 1$

$-65 \leq 65\sin(\theta + 22.6°) \leq 65$

$AC + BC + DE$ cannot be 70 cm
as the maximum length is 65 cm

(iv) Find the value of θ for which $AC + BC + DE = 50$ cm.

1.

$AC + BC + DE = 50$

$65\sin(\theta + 22.619°) = 50$

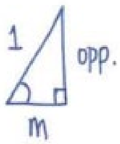
$\sin(\theta + 22.619°) = \frac{50}{65} = 0.76923$

$\theta + 22.619 = 50.284$

$\theta = 27.7°$

8 It is given that cos $A = -m$, where $m > 0$ and that A i obtuse.
Find the value of each of the fol lowing in terms of m.

(a) tanA [2]



$$\text{opp.} = \sqrt{1-m^2} \quad [M1]$$

$$\tan A = -\frac{\sqrt{1-m^2}}{m} \quad [A1]$$

(b) $\cot(180 - A) = \frac{1}{\tan(180-A)}$ [1]

$$= \frac{m}{\sqrt{1-m^2}} \quad [B1]$$

(c) $\cos\left(\frac{A}{2}\right)$ [3]

$$\cos A = 2\cos^2\frac{A}{2} - 1$$

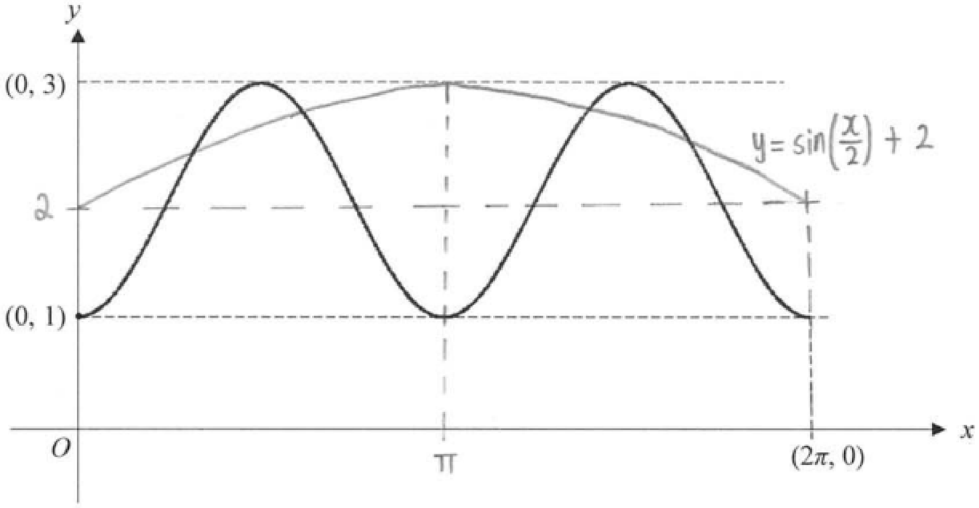
$$-m = 2\cos^2\frac{A}{2} - 1 \quad [M1]$$

$$\cos^2\frac{A}{2} = \frac{1-m}{2} \quad [M1]$$

$$\cos\frac{A}{2} = \sqrt{\frac{1-m}{2}} \quad \text{or} \quad -\sqrt{\frac{1-m}{2}} \text{ (rejected)}$$

[A1]

9



The diagram shows the curve $y = a + b\cos(cx)$ for $0 \le x \le 2\pi$.

(iv) Write down the value of a of b and of c. [3]

period = π

$\frac{2\pi}{c} = \pi$

$c = 2$ [B1]

$a = 2$ [B1]

$b = -1$ [B1]

(v) Sketch, on the same axes above, the graph of $y = \sin\left(\frac{x}{2}\right) + 2$ for $0 \le x \le 2\pi$. [3]

[1] shape
[1] period
[1] maximum.

period $= 2\pi \div \frac{1}{2}$
$= 4\pi$

$-1 \le \sin\left(\frac{x}{2}\right) \le 1$

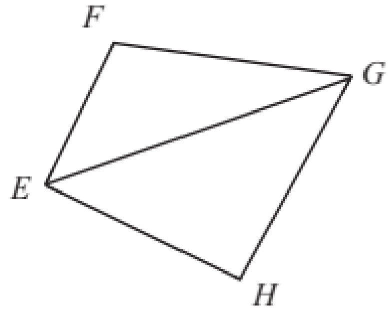
$1 \le \sin\left(\frac{x}{2}\right) + 2 \le 3$

(vi) Deduce the largest integer value of k such that $a + b\cos(cx) > \sin\left(\frac{x}{2}\right) + k$ for $0 \le x \le 2\pi$ [1]

$k = -1$ [B1]

10(i)	$d = \theta \sin\theta$ 5 cos θ	A2 [1 m for each term]
10(ii)	$d = R\sin(\theta - \alpha)$ $R = \sqrt{8^2 + 5^2} = \sqrt{89}$ $\tan\alpha = \frac{5}{8}$ $\alpha = 32.0°$ $d = \sqrt{89}\ \sin(\theta - 32.0°)$	A1 A1 A1
10(iii)	Max value of $d = \sqrt{89} = 9.43$ m when $\sin(\theta - 32.0°) = 1$ or $\theta = 122°$ Not possible for d to be 10 m	A1 A1
10(iv)	$\sqrt{89}\ \sin(\theta - 32.0°) = 6$ $\sin(\theta - 32.0°) = \frac{6}{\sqrt{89}}$ $\theta - 32.0° = 39.49°, 140.51°$ $\theta = 71.5°, 172.5°$	A1 , A1
11(i)	LHS $= \frac{\operatorname{cosec}^2\theta - 2}{\operatorname{cosec}^2\theta}$ $= 1 - \frac{2}{\operatorname{cosec}^2\theta}$ $= 1 - 2\sin^2\theta$ $= \cos 2\theta$	B1 B1 A1
11(ii)	$\cos 2\theta + 3\sin 2\theta = 0$ $3\sin 2\theta = -\cos 2\theta$ $\tan 2\theta = -\frac{1}{3}$ basic $\angle = 0.3218$ $2\theta = \pi - 0.3218, 2\pi - 0.3218, 3\pi - 0.3218$ $\theta = 1.41, 2.98, 4.55$	M1 M1 A2 Deduct 1 m for each wrong ans

12



EFGH is a plot of land that comprises two smaller plots, triangle *EFG* and triangle *EGH*. *EF* and *EH* are perpendicular, angle $FEG = \theta$, $EH = 42$ m, $EG = 55$ m and $EF = 48$ m.

(i) Show that the area, A m^2, of *EFGH* can be expressed as

$$A = 1320\sin\theta + 1155\cos\theta .$$ [2]

(ii) Express A in the form in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90$. [3]

(iii) Find the value of θ if the area is 1231 m^2. [2]

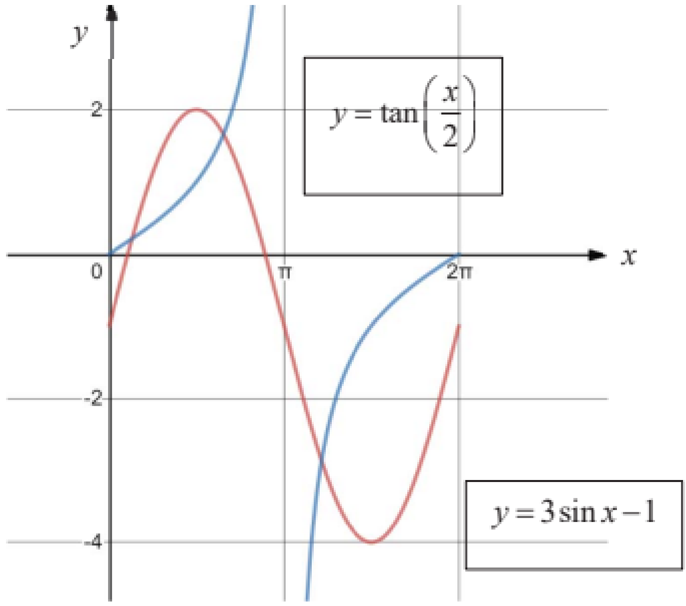
(i)	Area of triangle $EFG = \frac{1}{2} \times EF \times EG \sin\angle FEG$ $= \frac{1}{2} \times 48 \times 55 \sin\theta$ $= 1320\sin\theta$ Area of triangle $EGH = \frac{1}{2} \times EH \times EG \sin\angle FEG$ $= \frac{1}{2} \times 42 \times 55 \sin(90° - \theta)$ $= 1155(\sin 90° \cos\theta - \cos 90° \sin\theta)$ $= 1155\cos\theta$ $A =$ Area of triangle EFG + Area of triangle EGH $= 1320\sin\theta + 1155\cos\theta$	M1 – area of triangle M1 – use of trigonometry
(ii)	$1320\sin\theta + 1155\cos\theta = R\sin(\theta + \alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $R = \sqrt{1320^2 + 1155^2} = \sqrt{3076425}$ $\tan\alpha = \frac{1155}{1320}$, $\alpha = \tan^{-1}\left(\frac{1155}{1320}\right) = 41.1859°$ $A = \sqrt{3076425}\sin(\theta + 41.1859°)$ Or $A \approx 165\sqrt{113}\sin(\theta + 41.2°)$	M1 – finding R and α A1 A1

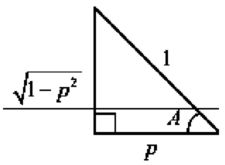
(iii)	$\sqrt{3076425}\sin(\theta+41.1859°)=1231$ $\sin(\theta+41.1859°)=\dfrac{1231}{\sqrt{3076425}}$ $\text{Basic angle}=\sin^{-1}\left(\dfrac{1231}{\sqrt{3076425}}\right)$ $=44.57449°$ $\theta+41.1859°=44.57449°,\ 180-44.57449°$ $\theta=3.3885°,\ 94.2397°(\text{NA})$ $\theta=3.4°$	M1 A1

13	Solve the equation $3\text{cosec}^2 x \sin x = 5(\cos x + \sin x)$, giving the principal values of *x*, in radians. [5]	
	$3\text{cosec}^2 x\ \sin x=5(\cos x+\sin x)$ $3\text{cosec}^2 x\ \sin x=5\cos x+5\sin x$ $3\text{cosec}^2 x=\dfrac{5\cos x+5\sin x}{\sin x}$ $=\dfrac{5\cos x}{\sin x}+\dfrac{5\sin x}{\sin x}$ $=5\cot x+5$ $3\text{cosec}^2 x=5\cot x+5$ $3(\cot^2 x+1)-5\cot x-5=0$ $3\cot^2 x+3-5\cot x-5=0$ $3\cot^2 x-5\cot x-2=0$ $(3\cot x+1)(\cot x-2)=0$ $\cot x=-\dfrac{1}{3}$ $\qquad$ $\cot x=2$ $\tan x=-3$ or $\tan x=\dfrac{1}{2}$ $x=-1.25$ $\qquad$ $x=0.464$	M1 – use of $\cot x=\dfrac{\cos x}{\sin x}$, $\text{cosec}\ x=\dfrac{1}{\sin x}$, $\tan x=\dfrac{\sin x}{\cos x}$ etc M1 – use identity to change into an equation with single trigonometric term M1 – reach quadratic equation and factorize, or use formula A2 – deduct one mark if more than the principal values are given.

14ai	Prove the identity $\sin x \cos x + \cot x \cos^2 x = \cot x$.	[4]
	LHS $= \sin x \cos x + \cot x \cos^2 x$ $= \cos x(\sin x + \cot x \cos x)$ $= \cos x\left(\sin x + \frac{\cos x}{\sin x}\cos x\right)$ $= \cos x\left(\sin x + \frac{\cos^2 x}{\sin x}\right)$ $= \cos x\left(\frac{\sin^2 x + \cos^2 x}{\sin x}\right)$ $= \cos x\left(\frac{1}{\sin x}\right)$ $= \cot x =$ RHS Or LHS $= \sin x \cos x + \cot x \cos^2 x$ $= \sin x \cos x + \frac{\cos x}{\sin x}\cos^2 x$ $= \frac{\sin^2 x \cos x}{\sin x} + \frac{\cos^3 x}{\sin x}$	[M1] [M1] [M1] [A1] [M1]

	$= \dfrac{\sin^2 x \cos x + \cos^3 x}{\sin x}$ $= \dfrac{\cos x\left(\sin^2 x + \cos^2 x\right)}{\sin x}$ $= \dfrac{\cos x\,(1)}{\sin x}$ $= \cot x = \text{RH}$	[M1] [M1] [A1]
aii	Hence, solve $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1$ for $0 \le x \le \pi$.	[3]
	Since $\sin x \cos x + \cot x \cos^2 x = \cot x$ Therefore, $\sin 3x \cos 3x + \cot 3x \cos^2 3x = \cot 3x$ and $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1 \quad \Rightarrow \quad \cot 3x = 1$ $0 \le x \le \pi$ $0 \le 3x \le 3\pi$ Let the basic angle be α $\tan \alpha = 1$ $\alpha = \dfrac{\pi}{4}$ $3x = \dfrac{\pi}{4}, \pi + \dfrac{\pi}{4}, 2\pi + \dfrac{\pi}{4}$ $3x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}$ $x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{3\pi}{4}$ rad	[M1] [M1] [A1]
bi	On the same axes, sketch the graphs of $y = 3\sin x - 1$ and $y = \tan\dfrac{x}{2}$ for $0 \le x \le 2\pi$.	[5]
		For tan curve, [B1] for shape, [B1] for asymptote For Sine curve, [B1] for shape [B1 for correct max and

		minimum values [B1] for correct period for both graphs [subtract 1mark for wrong or no labels for axes or functions]
bii	Hence, state the number of solutions of $3\sin x - 1 = \tan\frac{x}{2}$ for $0 \le x \le 2\pi$.	[1]
	From the sketch, the two functions intersect at three points. Hence there are three solutions for the equation for $0 \le x \le 2\nu$	[B1]

15	Given that cos $A = p$ and that A is acute, express the following in terms of p.	
i	sin 2A	[3]
		M1 for getting the length of opposite side

	$\sin 2A = 2\sin A\cos A$ $= 2p\sqrt{1-p^2}$ Or $\cos^2 A + \sin^2 A = 1$ $\sin^2 A = 1 - \cos^2 A$ $\sin^2 A = 1 - p^2$ $\sin A = \sqrt{1-p^2}$ (reject negative as A is acute) $\sin 2A = 2\sin A\cos A$ $= 2p\sqrt{1-p^2}$	M1 A1 M1 M1 A1

ii	$\cos\frac{A}{2}$	[3]
	$\cos A = 2\cos^2\frac{A}{2} - 1$ $\cos\frac{A}{2} = \pm\sqrt{\frac{\cos A + 1}{2}}$ $\cos\frac{A}{2} = \pm\sqrt{\frac{p+1}{2}}$ $\cos\frac{A}{2} = -\sqrt{\frac{p+1}{2}}$ (rejected) or $\cos\frac{A}{2} = \sqrt{\frac{p+1}{2}}$	M1 M1 for rejection A1

16 Given that θ is obtuse and $\tan\theta = a$, express, in terms of a,

(i) $\cos\theta$, [3]

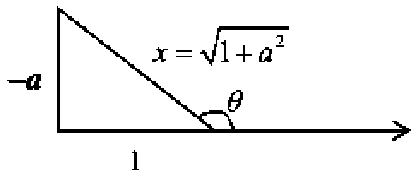
(ii) $\operatorname{cosec}\theta$. [2]

Solution:

(i) Since $\tan\theta = a$ and θ is obtuse, $a < 0$. **M1**

$x = \sqrt{1+a^2}$ **M1**

$\cos\theta = -\frac{1}{\sqrt{1+a^2}}$ **A1**



(ii) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$= \frac{1}{\frac{-a}{\sqrt{1+a^2}}}$ **M1**

$= -\frac{\sqrt{1+a^2}}{a}$ **A1**

17 It is given that $f(x) = 2\sin\frac{x}{2}$ and $g(x) = 3\cos x + 1$ where $0 \le x \le 2\pi$.

(i) State the period of $f(x)$. [1]

Solution:

Period = 4π **B1**

(ii) State the smallest value of $f(x)$. [1]

Solution:

Smallest value of $f(x)$ is 0. **B1**

(iii) State the largest value of $g(x)$. [1]

Solution:

Largest value of $g(x)$ is 4. **B1**

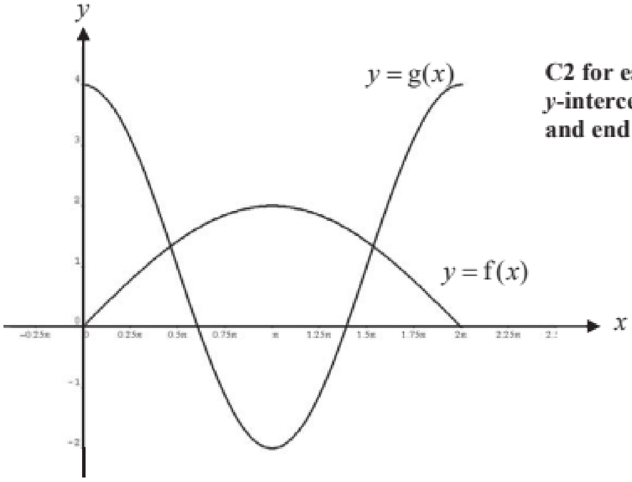
(iv) State the largest value of $|f(x) - g(x)|$. [1]

Solution:

Largest value of $|f(x) - g(x)|$ is 4 **B1**

(v) Sketch, on the same axes, the graphs of $y = \text{f}(x)$ and $y = \text{g}(x)$ for $0 \le x \le 2\pi$. [4]

Solution:



C2 for each graph (correct *y*-intercept, turning points and end points)

(vi) Given that the solutions to the equation $\text{f}(x) = \text{g}(x)$ for $0 \le x \le 2\pi$ are a and b where $a < b$, state the range of value of x for which $\text{f}(x) \ge \text{g}(x)$. [1]

Solution:

$\text{f}(x) \ge \text{g}(x)$ when $\underline{\underline{a \le x \le b}}$ **B1**

18 **(a)** It is given that $\tan(A + B) = 8$ and $\tan B = 2$. **Without using a calculator**, find the exact value of $\cot A$. [3]

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Since $\tan(A+B) = 8$ and $\tan B = 2$,

$$8 = \frac{\tan A + 2}{1 - 2\tan A}$$ **[M1]**

$$8 - 16\tan A = \tan A + 2$$
$$17\tan A = 6$$

$$\tan A = \frac{6}{17}$$ **[M1]**

$$\cot A = \frac{17}{6}$$ **[A1]**

(b) **(i)** Prove that $\sin 2x(\cot x - \tan x) = 2\cos 2x$. [3]

$\text{LHS} = \sin 2x(\cot x - \tan x)$

$= 2\sin x \cos x\left(\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\right)$ **[M1]**

$= 2(\cos^2 x - \sin^2 x)$ **[M1]**

$= 2\cos 2x$ **[A1]**

$= \text{RHS}$ (proven)

(ii) Hence solve the equation $\sin 2x(\cot x - \tan x) = \sec 2x$ for $0 \le x \le \pi$.

[4]

$$\sin 2x(\cot x - \tan x) = \sec 2x$$

$$2\cos 2x = \frac{1}{\cos 2x}$$

$$\cos^2 2x = \frac{1}{2}$$ **[M1]**

$$\cos 2x = \pm\sqrt{\frac{1}{2}}$$ **[M1]**

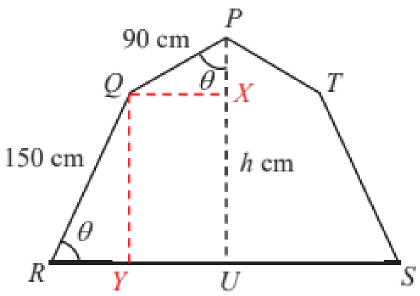
Basic angle = $\frac{\pi}{4}$ rad

$$2x = \frac{\pi}{4}, \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$$

$$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$ **[M1]**

$$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$$ **[A1]**

19 The diagram shows the side view *PQRST* of a tent. The tent rests with *RS* on horizontal ground. *PQRST* is symmetrical about the vertical *PU*, where *U* is the midpoint of *RS*. Angle QPU = angle $QRU = \theta$ radians and the lengths of *PQ* and *QR* are 90 cm and 150 cm respectively. The vertical height of *P* from the ground is *h* cm.



(i) Explain clearly why $h = 90\cos\theta + 150\sin\theta$. [2]

Let the foot of the perpendicular from *Q* to *PU* and *RU* be *X* and *Y* respectively.

$PX = 90\cos\theta$
$QY = 150\sin\theta$ } **[M1 for both]**

$h = PX + QY$

$= 90\cos\theta + 150\sin\theta$ **[A1]**

(ii) Express *h* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

$R\cos\alpha = 90$

$R\sin\alpha = 150$ **[M1]**

$\tan\alpha = \dfrac{150}{90}$ **[M1]**

$\alpha = 1.0304$

$R = \sqrt{90^2 + 150^2}$

$= 30\sqrt{34}$ **[M1]**

$h = 30\sqrt{34}\cos(\theta - 1.03)$ **[A1 - accept if $R = 175$ (3s.f.)]**

(iii) Find the greatest possible value of h and the value of θ at which this occurs. [3]

Greatest value of h occurs when $\cos(\theta - 1.03) = 1$ **[M1]**

Greatest value of $h = 30\sqrt{34}$ **[A1 – Accept 175]**

when $\theta = 1.03$. **[A1]**

(iv) Find the values of θ when $h = 160$. [3]

$30\sqrt{34}\cos(\theta - 1.0304) = 160$

$\cos(\theta - 1.0304) = \dfrac{160}{30\sqrt{34}}$ **[M1]**

Basic angle = 0.41613

$\theta - 1.0304 = -0.41613$ or 0.41613

$\theta = 0.614$ or 1.45 (to 3s.f.) **[A1, A1]**

20(i)	$\dfrac{\tan^2 x - 1}{\tan^2 x + 1}$ $= \dfrac{\frac{\sin^2 x}{\cos^2 x} - 1}{\frac{\sin^2 x}{\cos^2 x} + 1}$ $= \dfrac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x}$ $= \sin^2 x - \cos^2 x$ $= 1 - \cos^2 x - \cos^2 x$ $= 1 - 2\cos^2 x$ (shown)

(ii)	$1-2\cos^2 x=\frac{1}{2}$ $\cos^2 x=\frac{1}{4}$ $\cos x=\pm\frac{1}{2}$ $\alpha=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$ $x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3}$