Name:	Target Grade:	Actual Grade:

BRIGHT CULTURE

DIFFERENTIATION

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Find a quiet, comfortable spot free place from distractions.
2. Spend one minute on each mark.
3. Time yourself for every single question.
4. Every chapter has their own question types. Ensure that you know the different question type for each chapter.
5. Make a conscientious effort to remember your mistakes, especially in terms of answering techniques. E.g Take a picture for the mistakes that you made, keep it in a photo album, and revise it over and over again.
6. Highlight question types that you tend to keep making mistakes and review them nearing exams.
7. Always review the common questions and question type that you tend to make mistakes nearing exams.
8. During exams, classify the question type and recall what you have learnt, how you need to analyse the questions for the different question type, what you need to take note of and answer with the correct answering techniques!

✨ Wishing you all the best for this test!

You've got this!

💡 With lots of love,
Bright Culture 💛

MARKS

If you are struggling in this paper, means you need to work harder!

If you need any professional guidance and further advice on how to advance, feel free to WhatsApp us at 91870820 or find us at [www.bright-culture.com/.](http://www.bright-culture.com/) We are committed to connect you to your future to reach your goals.

CHAPTER 9: DIFFERENTIATION

1 The equation of a curve is $y = \ln\left(\frac{x-3}{x+6}\right)$ for $x > 3$. Determine whether y is an increasing or decreasing function. [5]

2 The function f (x) is defined by $f(x) = \frac{5x+2}{2x-3}$ for all values of x, $x \neq \frac{3}{2}$

Determine, with working, whether f(x) is an increasing function or a decreasing function. [3]

3 The function f is defined by $\mathrm{f}(x) = x\mathrm{e}^{-\frac{x}{5}} + k\mathrm{e}^{3x}$, where k is a constant.
Given that $\mathrm{f}'(0) = 5$, find the value of k. [4]

4 (a) It is given that $f(x) = \ln \sqrt[3]{\frac{5+x}{5-x}}$.

(i) Find $f'(x)$ and $f''(x)$. [4]

(ii) Hence determine the range of values of x for which both $f'(x)$ and $f''(x)$ are positive. [4]

(b) Show that $\frac{d}{dx}\left[4\sin^2\left(\frac{x}{2}+\pi\right)\right]=k\sin x$ where k is a constant. [3]

5 Given that $y = \operatorname{cosec} x \tan x$,

(i) show that $\dfrac{dy}{dx} = \sin x \sec^2 x$, and [2]

(ii) determine where y is decreasing for $0 \le x \le 2\nu$. [2]

ANSWERS

1 The equation of a curve is $y = \ln\left(\frac{x-3}{x+6}\right)$ for $x > 3$. Determine whether y is an increasing or decreasing function. [5]

$$y = \ln\left(\frac{x-3}{x+6}\right)$$
$$= \ln(x-3) - \ln(x+6)$$

(Simplifying using the laws of logarithms will reduce the complexity of the technique of differentiation.)

$$\frac{dy}{dx} = \frac{1}{x-3} - \frac{1}{x+6}$$
$$= \frac{x+6-(x-3)}{(x-3)(x+6)}$$
$$= \frac{9}{(x-3)(x+6)}$$

[Note: $\frac{d}{dx}(\ln u) = \frac{u'}{u}$]

Alternative Method

$$\frac{dy}{dx} = \frac{\frac{(x+6)(1)-(x-3)(1)}{(x+6)^2}}{\left(\frac{x-3}{x+6}\right)}$$

This is NOT recommended!

$$= \frac{x+6-x+3}{(x+6)^2} \times \frac{x+6}{x-3}$$

$$= \frac{9}{(x-3)(x+6)}$$

For $x > 3$, 9 is a positive constant, $x - 3$ and $x + 6$ are both positive.

Since $\frac{dy}{dx} > 0$. Hence y is an increasing function.

2 $f'(x) = \frac{-19}{(2x-3)^2}$ Decreasing Function

3 $k = \frac{20}{3}$

4(a)(i)	$f(x)=\ln\left(\frac{5+x}{5-x}\right)^{\frac{1}{3}}$ $=\frac{1}{3}\ln\left(\frac{5+x}{5-x}\right)$ $=\frac{1}{3}\left[\ln(5+x)-\ln(5-x)\right]$ $f'(x)=\frac{1}{3}\left(\frac{1}{5+x}+\frac{1}{5-x}\right)$ $=\frac{1}{3}\left[\frac{5-x+5+x}{(5+x)(5-x)}\right]$ $=\frac{10}{3(5+x)(5-x)}$ $=\frac{10}{3\left(25-x^2\right)}$ $f''(x)=\frac{10}{3}(-1)\left(25-x^2\right)^{2}(-2x)$ $=\frac{20x}{3\left(25-x^2\right)^2}$	M1 M1 A1 A1
4(a)(ii)	For $f'(x)>0$ $25-x^2>0$ $(5+x)(5-x)>0$ $-5<x<5$ For $f''(x)>0$ $20x>0$ $x>0$ For both $f'(x)$ and $f''(x)$ to be positive, $0<x<5$	M1 A1 A1 A1
4(b)	$\frac{d}{dx}\left[4\sin^2\left(\frac{x}{2}+\pi\right)\right]$ $=4\times2\sin\left(\frac{x}{2}+\pi\right)\cos\left(\frac{x}{2}+\pi\right)\times\frac{1}{2}$ $=4\sin\left(\frac{x}{2}+\pi\right)\cos\left(\frac{x}{2}+\pi\right)$ $=2\sin2\left(\frac{x}{2}+\pi\right)$ $=2\sin(x+2\pi)$ or $2(\sin x\cos2\pi+\cos x\sin2\pi)$ $=2\sin x$	M1 M1 A1

5	Given that $y = \operatorname{cosec} x \tan x$, (i) show that $\frac{dy}{dx} = \sin x \sec^2 x$. [2] (ii) determine where y is decreasing for $0 \le x \le 2\pi$. [2]	
(i)	$y = \operatorname{cosec} x \tan x$ $= \frac{1}{\sin x} \times \frac{\sin x}{\cos x}$ $= \frac{1}{\cos x}$ $= (\cos x)^{-1}$	M1
	$\frac{dy}{dx} = (-1)(\cos x)^{-2}(-\sin x)$ $= \frac{1}{\cos^2 x}(\sin x)$ $= \sec^2 x \sin x$	M1 – chain rule and differentiation of trigonometric functions
	Or $y = \operatorname{cosec} x \tan x$ $= \frac{1}{\sin x} \times \tan x$ $= (\sin x)^{-1} \tan x$ $\frac{dy}{dx} = \frac{1}{\sin x} \cdot \sec^2 x + \tan x\left(-\sin^{-2} x \cos x\right)$	
	$= \frac{1}{\sin x} \cdot \sec^2 x - \frac{\sin x}{\cos x}\left(\frac{\cos x}{\sin^2 x}\right)$	M1
	$= \frac{1}{\sin x} \cdot \sec^2 x - \frac{1}{\sin x}$ $\frac{dy}{dx} = \frac{\sec^2 x - 1}{\sin x}$ $= \frac{\tan^2 x}{\sin x}$	
	$= \frac{1}{\sin x}\left(\frac{\sin^2 x}{\cos^2 x}\right)$	M1
	$= \frac{1}{\cos^2 x}(\sin x)$ $= \sin x \sec^2 x$	

	$y = \operatorname{cosec} x \tan x$ $= \dfrac{\tan x}{\sin x}$ $\dfrac{dy}{dx} = \dfrac{\sin x \sec^2 x - \tan x \cos x}{\sin^2 x}$ $= \dfrac{\sin x \sec^2 x - \sin x}{\sin^2 x}$ $= \dfrac{\sin x\left(\sec^2 x - 1\right)}{\sin^2 x}$ $= \dfrac{\sin x\left(\tan^2 x\right)}{\sin^2 x}$ $= \dfrac{1}{\sin x}\left(\dfrac{\sin^2 x}{\cos^2 x}\right)$ $= \dfrac{1}{\cos^2 x}(\sin x)$ $= \sin x \sec^2 x$	M1 M1
(ii)	For a decreasing function, $\dfrac{dy}{dx} < 0$ For $0 \le x \le 2\pi$, $\sec^2 x > 0$, $x \ne \dfrac{\pi}{2}, \dfrac{3\pi}{2}$, $\sin x < 0$ for $\pi < x < 2\pi$ Hence $\dfrac{dy}{dx} < 0$ for $\pi < x < 2\pi$, $x \ne \dfrac{3\pi}{2}$	B1 B1 - if didn't write $x * \dfrac{3\pi}{2}$, its acceptable.